Exercise 1. Consider the following game. You start by rolling one die and it comes up value $D$, and then you flip $D$ coins. Your final result is $X = 2H$ where $H$ is the number of heads that you flip.

- Find $E[D], E[X]$.
- Find $E[X/D]$.
- What is the probability that $X$ is odd?
- Given that $D = 1$ what is the probability that $X = 0$?
- Given that $X = 0$ what is the probability that $D = 1$?
- Which is larger, $\text{Var}[D]$ or $\text{Var}[X]$ (try answering without computing and then computing to verify your guess)?

Exercise 2. Suppose we play the following variant of Let’s Make a Deal. Suppose there are four doors and two prizes put behind different doors. The prizes are worth $500 and $1000 respectively. There is nothing behind the other two doors. Assume that the game setters choose the doors to put the prizes behind randomly. You get to choose one door. From watching the show ahead of time, you have seen that the host always selects one door that is not selected by the player and does not contain a prize and opens it. The host never opens the fourth door unless she has to. If two of the first three doors have no prize and have not been chosen by the player, she will randomly choose which of those two to open. After she opens the door, you may change your choice. Under each of the following scenarios, give your best strategy and your conditional expected winnings.

- You chose door 1 and the host opened door 2.
- You chose door 1 and the host opened door 4.
- You chose door 4 and the host opened door 2.

Exercise 3. As part of the freshman orientation at the University of Chicago, the new students are tested to see if they have extrasensory perception (ESP). Each student is asked ten consecutive true-false questions about a subject they know nothing about. However, there is another person in the room who does know the answer and thinks the answer as hard as she can. After administering the test to all the students, there were two students who got all 10 questions right. Given this, which of the following should you conclude:

- These students have ESP and were able to determine the answer from the brainwaves of the woman who knew the answer.
• These students cheated on the test.
• Nothing unexpected happened.

You may assume that the entering class at University of Chicago has 1400 students.

**Exercise 4.** You flip a coin until the first time you have flipped three heads in a row. Find the expected number of flips.

**Exercise 5.** You have managed to convince a very rich person to play the following game. You roll two dice. If the sum is 7, he gives you $1. If the sum is 8, you give him $1. If anything else comes up, no money exchanges hands. You have only $5 and will keep playing the game as long as you still have money. Assuming your opponent never quits, what is the probability that you will never go bankrupt? (Hint: let \( q(k) \) be the probability of going bankrupt given that you currently have \$k. Write an equation for \( q(k) \).)

**Exercise 6** (From Quantnet forum). Suppose two players play a game. Player A flips coins until she gets two heads in a row. Player B flips coins until he gets three heads in a row. Player B wins if he flips fewer coins than Player A. What is the probability that Player B wins? (Hint: assume they flip at the same time. Let \( q(j, k) \), \( j = 0, 1, 2, k = 0, 1 \) be the probability that Player B wins given that currently A has \( j \) consecutive heads and B has \( k \) consecutive heads. Write equations for \( q(j, k) \).)